Structural Equation Modeling: Advantages, Challenges, and Problems

Structural equation models (SEM) are complex methods of data analysis. In the social sciences, they allow for analyses that would not be possible using other methods. Even in cases where alternative methods of analyses are available, structural equation modeling may offer more meaningful and more valid results.

On the other hand, more effort is necessary until the greater complexity pays off. Assumptions on the data may be higher, and the process of interpreting the results is more complex compared to other methods of data analysis.

1 Advantages

Practical advantages of using structural equation modeling for data analysis include:

- **Validity:** Theories in the social sciences frequently refer to variables that can not directly be observed (*constructs*), but that can only be inferred from observable variables (*indicator variables*). To operationalize these constructs, often many different variables come into consideration, and none of them may provide an optimal operationalization on its own. Structural equation modeling allows to make use of several indicator variables per construct simultaneously, which leads to more valid conclusions on the construct level. Using other methods of analysis would often result in less clear conclusions, and/or would require several separate analyses.

- **Reliability/Measurement Error:** Data in the social sciences frequently contain a non-neglible amount of measurement error. Structural equation modeling can take measurement error into account by explicitly including measurement error variables that correspond to the measurement error portions of observed variables. Therefore, conclusions about relationships between constructs are not biased by measurement error, and are equivalent to relationships between variables of perfect reliability.

- **Complex Models:** Theories in the social sciences frequently involve complex patterns of relationships or differences between a multitude of variables, conditions or groups. Structural equation modeling allows to model and test complex patterns of relationships, including a multitude of hypotheses simultaneously as a whole (including mean structures and group comparisons). Using other methods of analysis, this would frequently require several separate analyses.

- **Confirmatory Approach:** For hypotheses testing, simple statistical procedures usually provide tests on the basis of explained variance in single criterion variables. This is inappropriate for evaluating complex models containing a multitude of variables and relationships. In contrast, structural equation modeling allows to test complex models for their compatibility with the data in their entirety, and allows to test specific assumptions about parameters (e. g., that they equal zero, or that they are identical to each other) for their compatibility with the data. In doing so, the variances and covariances of all the observed variables are factored in systematically: The empirical relationships between all observed variables (empirical covariance matrix) are compared to the relationships implied by the structure of the theoretical model (model-implied covariance matrix). This allows for:

  – Global assessment: The model fits the data well or not so well.
  – Local assessment: The model is or is not able to correctly reproduce relationships between particular variables. This can point to specific areas/parts where the model may be deficient.
  – Exploratory suggestions for potential model improvements (modification indices): These suggestions can then be evaluated for interpretability and compatibility with an underlying theory.

2 Challenges and Potential Problems

The complexity of structural equation modeling comes with statistical and interpretational challenges and potential problems:

- **Model Identification/Parameter Identification:** In structural equation models, a multitude of parameters (path coefficients, factor loadings, variances, etc.) corresponding to various hypotheses are estimated simultaneously (so that the empirical relationships between the observed variables can be reproduced by the model as good as possible). This only works if the empirical data provide enough information to estimate all these parameters.

  Most often, structural equation modeling is not based on raw data as input information, but on the empirical covariances of all indicator variables. Therefore, it is not possible to estimate more model parameters than there are (distinct) entries in the empirical covariance matrix. Given k indicator variables, a maximum of $k(k+1)/2$ parameters can be estimated (then, the model would be *just identified*). Hypotheses testing is only possible as long as there are *less* parameters to be estimated than there are distinct empirical covariances, i. e. less than $k(k+1)/2$ (the model would then be *overidentified*).

  This global condition for model identification is necessary, but not sufficient. It can happen that, despite a satisfied global condition, certain parts of the model are not identified (e. g., when empirical relationships between variables are particularly weak). Possible remedies or workarounds include reformulating the model, incorporating additional variables, or testing identified model parts separately.

- **Estimation Methods and Estimation Problems:** Simultaneously including a multitude of relationships is computationally intensive and is being done by iterative algorithms, i. e. by trying to gradually approach an optimal solution (in terms of reproducing the empirical relationships). This can lead to estimation problems:

  1. The algorithm may not converge, i. e. no optimal solution can be found.
  2. The algorithm may converge and result in a supposedly optimal solution, but the parameter estimates do not make sense (so-called Heywood cases). For example, negative estimates of variances may occur, despite the fact that empirical variances can not be negative.

  This tends to happen mostly in situations where assuptions of the respective method of estimation are violated (see below), and/or in cases where the model analyzed is based on wrong assumptions or hypotheses (*misspecified* model). Possible remedies or workarounds include the use of a different estimation method (e. g., one with less strict assumptions), simplifying the model, separate analyses of parts of the model, and/or a larger sample size.

- **Assumptions—Sample Size and Distributions:** Most often, parameter estimation in structural equation models is done by maximum likelihood, which is based on certain assumptions:

  1. Multivariate normal distribution of the indicator variables
  2. Large sample size

  In practice, data are rarely ever multivariate normal, and often they are univariate nonnormal already. In addition, especially in psychology, samples frequently consist of a few hundred cases at best, while the mathematical foundation of maximum likelihood estimation is based on asymptotically large sample sizes (going to infinity).

  In particular, combining small sample sizes, nonnormal data, and weak empirical relationships between variables can lead to estimation problems and unreliable results. Potential remedies include certain correction factors (e. g., for standard error estimates in cases of nonnormal data), or the use of a different estimation method (e. g. Unweighted Least Squares; explained in more detail on the handout on estimation methods and distributional assumptions, "Parameter-Schätzmethoden und Verteilungsvoraussetzungen" [sorry, presently in German only]).

- **Interpretation of Results:** Properly assessing the quality of a structural equation model is usually not the matter of a single test of significance or a single measure of explained variance, but regularly comprises several steps and several coefficients or tests.

  1. Examining the parameter estimates for plausibility (to rule out Heywood cases which would call any further interpretation into question)
  2. Assessing the global "model fit", i. e. the global match between the empirical covariance matrix of all indicator variables, and the covariance matrix reproduced by the model: For this purpose, there is a χ^2 test for significant differences between model and data, and a multitude of descriptive goodness-of-fit indices as measures of either

the degree of congruence between model and data, or discrepancies between model and data (see the handout "Recommendations for Model Evaluation: Some Rules of Thumb").

3. Assessing particular aspects of model fit, especially by checking the (standardized) residuals (differences between empirical and model-implied covariances), which provide evidence whether the empirical covariances between specific variables can or can not be correctly reproduced by the model (pointing to possible reasons for poor global model fit)
4. If model fit is satisfactory: Interpreting the parameter estimates, testing parameters for significance, assessing the predictive accuracy of the model (variance explained in the individual variables)

Structural equation modeling is often employed as a statistical means to test causal hypotheses. However, decision problems can occur in cases when there are two or more alternative models which make fundamentally different assumptions about the variables' causal relationships, but still lead to the exact same model fit, making it impossible to base a decision solely on statistical criteria.

3 Online Material

Current versions of our handouts are available online:
`http://user.uni-frankfurt.de/~cswerner/sem/`

The free LISREL Student Version for teaching purposes (Windows) can be downloaded from Scientific Software here:
`http://www.ssicentral.com/lisrel/student.html`

The Mac and Linux versions are not linked from the above page any longer, but at least the Linux version can be found here:
`ftp://ftp.lisrel.com/lisrel/linux/student/`

In contrast to the full commercial version, the student version is restricted to 15 manifest variables, which is sufficient for basic training and exercises.

4 Recommended Reading

Kline, R. B. (1998). *Principles and practice of structural equation modeling.* New York: Guilford. (accessible, up-to-date introduction to structural equation modeling; the current second edition (2005) generally is as easily accessible as the first, but more oriented towards the AMOS software, while the first edition is more oriented towards LISREL; in addition, the first also offers a bit more detail in certain areas)

Bollen, K. A. (1989). *Structural equation modeling with latent variables* (2nd ed.). New York: Wiley. (still one of the most comprehensive sources on structural equation modeling)